Simulation of an AR(1) model

In this short note, we show how to simulate from an AR(1) model. In particular, we shall simulate realizations from the model given by

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.t(v), \text{ and } |\beta_1| < 1$$

where $t(v)$ denotes the student distribution with v degree of freedom. Note that if $\epsilon_t \sim t(v)$, then $\sigma^2 = \text{Var}(\epsilon_t) = \frac{v}{v-2}$. The statistical properties of the model are as follows,

$$E(y_t) = \frac{\beta_0}{1-\beta_1}, \quad \text{Var}(y_t) = \frac{\sigma^2}{1-\beta_1^2}, \quad \text{Corr}(y_t, y_{t-j}) = \beta_1^j.$$

Let us check these results empirically.

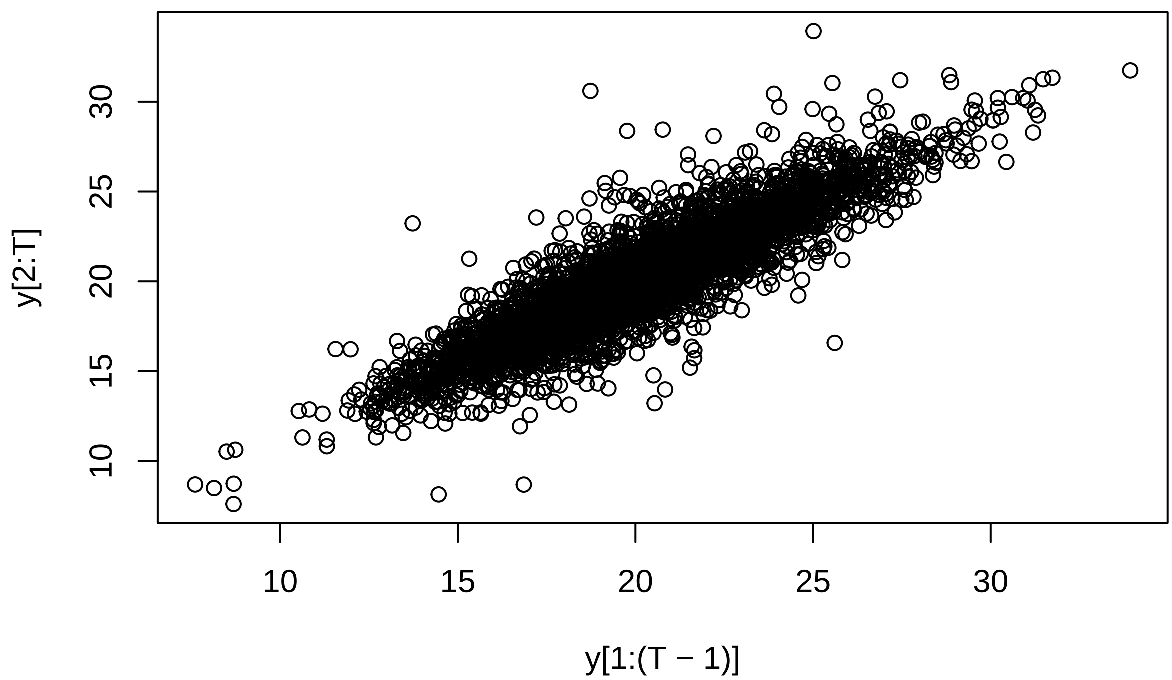
```
T = 5000 ## sample size
beta_0 = 2
beta_1 = 0.9
unc_mean = beta_0/(1-beta_1)
df = 5
y = numeric(T)
y[1] =unc_mean
for(t in 2:T){
  y[t] = beta_0 + beta_1*y[t-1] + rt(1,df)
}
stat_th = c(unc_mean,(df/(df-2))/(1-beta_1^2),beta_1,beta_1^2)

corr_emp = mean((y[2:T]-mean(y))*(y[1:(T-1)]-mean(y)))/var(y)
corr_emp2 = mean((y[3:T]-mean(y))*(y[1:(T-2)]-mean(y)))/var(y)
stat_emp = c(mean(y),var(y),corr_emp,corr_emp2)

df_stat = data.frame(cbind(stat_th,stat_emp))
rownames(df_stat) = c("Expectation","Variance","1-lag Correlation","2-lag Correlation")
colnames(df_stat) = c("Theoretical","Empirical")
(df_stat)
```

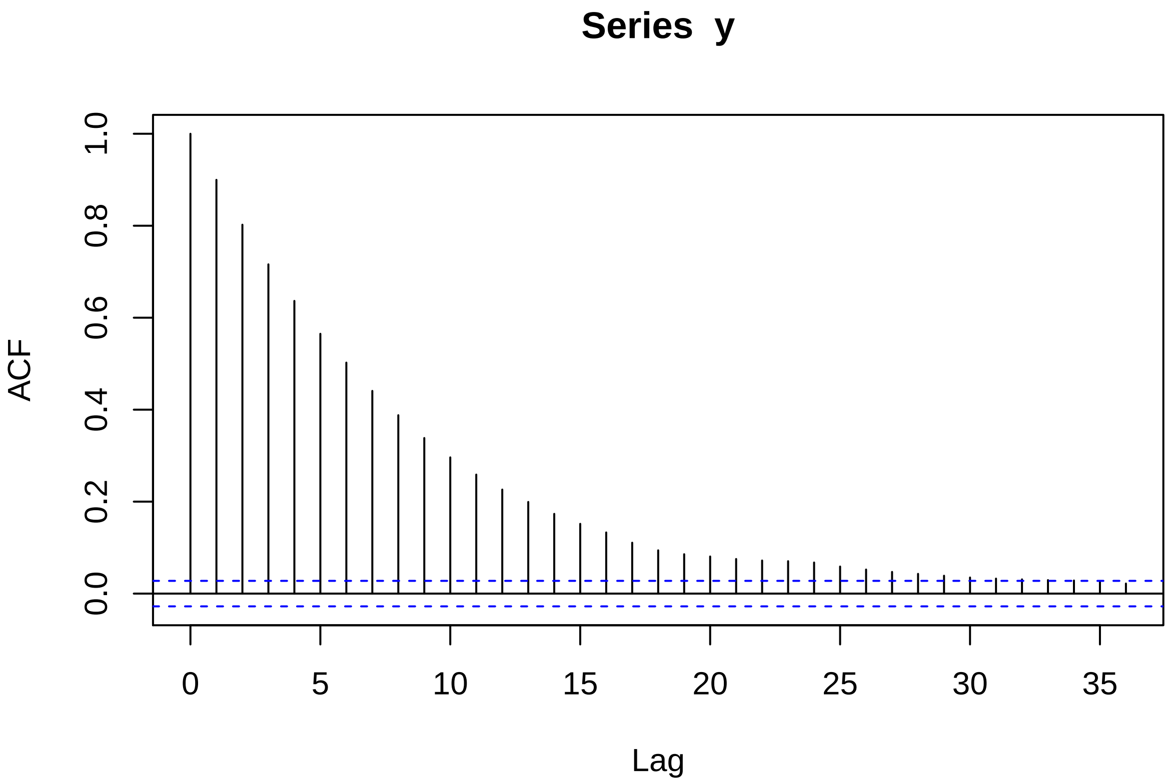
```
##                     Theoretical  Empirical
## Expectation            20.00000 20.1788008
## Variance                8.77193  9.7425194
## 1-lag Correlation       0.90000  0.8998014
## 2-lag Correlation       0.81000  0.8024396
```

```
plot(y[1:(T-1)],y[2:T])
```



We observe that the empirical statistics are close to the theoretical one. We conclude that we have correctly simulated the AR(1) process. We end this exercise by computing the autocorrelation function (ACF). The empirical autocorrelation should decay geometrically.

```
acf(y)
```


Series y
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
Lag