Autocorrelation test

Autocorrelation

In this short note, we apply the Box-Pierce tests and check for autocorrelations in two time series. We shall do the following:

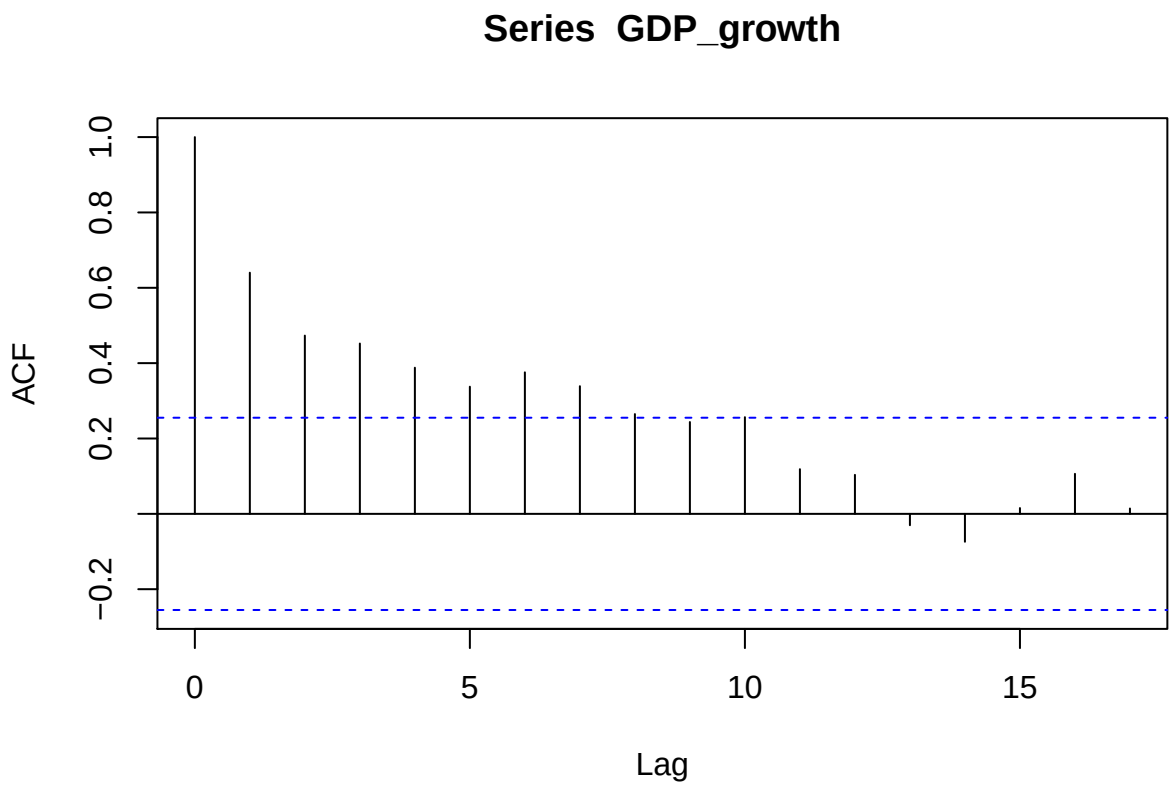
1. Compute the autocorrelation function using R.
2. Compute the Box-Pierce test using R and manually.

To illustrate this test, we focus on two series that are the GDP of France and Bitcoin returns.

GDP growth rate

We first load the yearly GDP growth rate and check its autocorrelation values.

```
url = "https://raw.githubusercontent.com/adufays/GDP_expectancy/main/france-gdp-growth-rate.csv"
data = read.csv(url, sep=",")
GDP_growth = data[,2]
autocorr = acf(GDP_growth)
```



```
(autocorr)
```

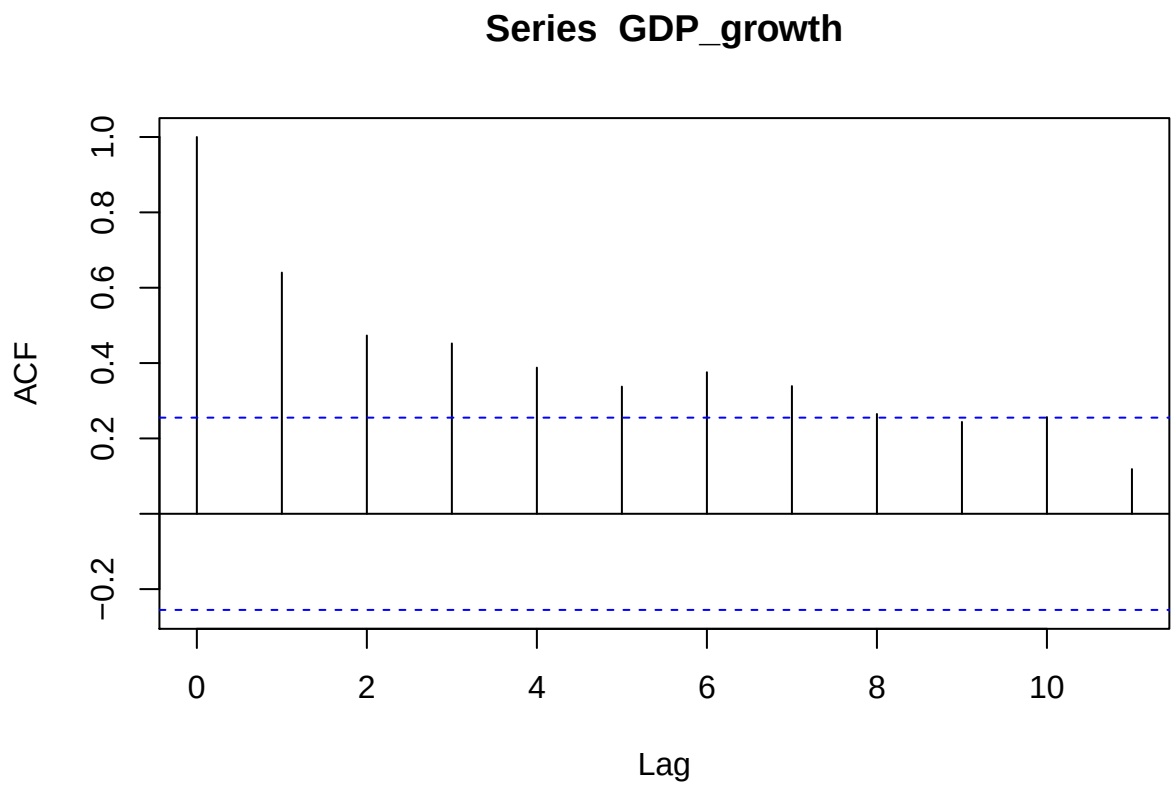
```
##
## Autocorrelations of series 'GDP_growth', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.640  0.473  0.452  0.388  0.338  0.376  0.339  0.265  0.244  0.257
##     11     12     13     14     15     16     17
##  0.119  0.104 -0.030 -0.074  0.015  0.106  0.014
```

We observe the presence of autocorrelation and that the 1-lag autocorrelation is the highest autocorrelation value. Using the Box-Pierce statistical test, we formally verify the presence of autocorrelation up to 10 lags.

```
Box.test(GDP_growth,lag = 10) ## R function
```

```
##
##  Box-Pierce test
##
## data:  GDP_growth
## X-squared = 91.673, df = 10, p-value = 2.442e-15
```

```
## Performing manually the same test
max_lag = 11
acf_val = acf(GDP_growth,lag.max = max_lag)
```




```
val = acf_val$acf
T = length(GDP_growth)
T*sum(val[2:max_lag]^2)
```

```
## [1] 91.67323
```

```
qchisq(0.95,max_lag-1)
```

```
## [1] 18.30704
```

We reject the Null hypothesis of no-autocorrelation in the series because the critical value amounts to 18.3 while the value of the test equals to 91.67.

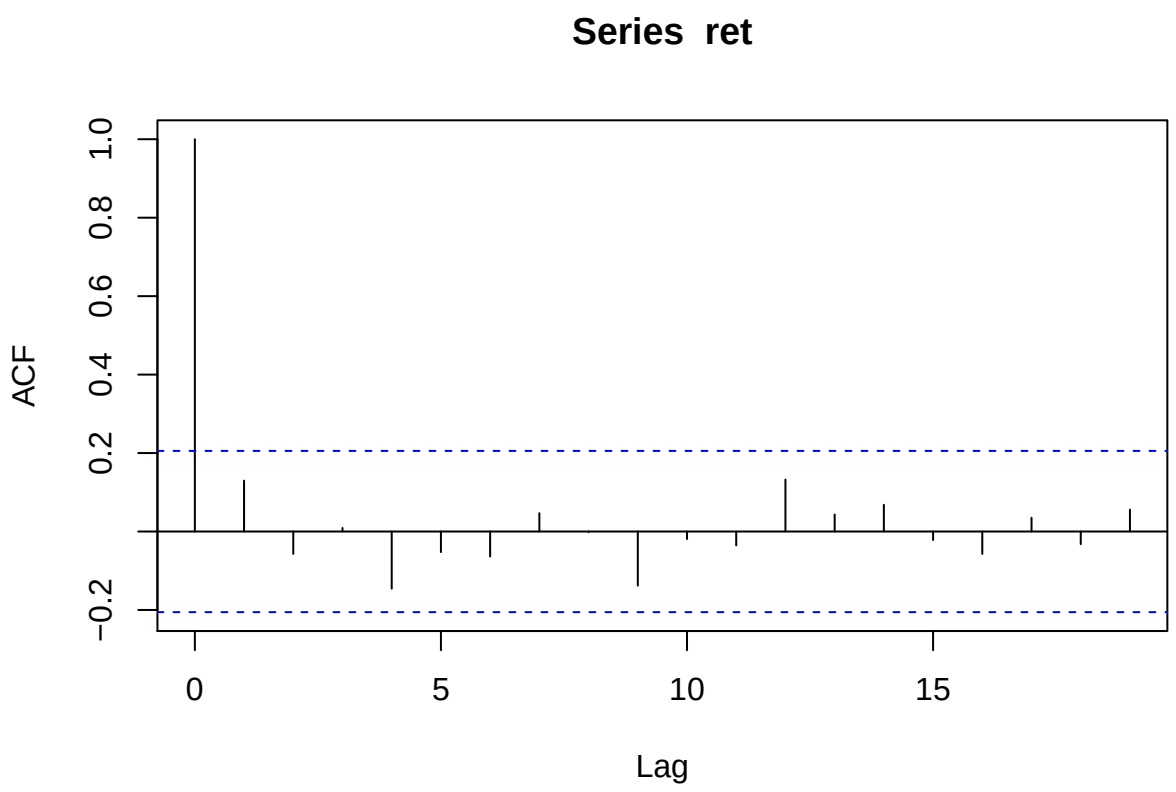
Bitcoin returns

First, we load the BTC prices and compute the returns in percentage.

```
url = "https://raw.githubusercontent.com/adufays/GDP_expectancy/main/DATA_BT.csv"
DATA = read.csv(url)
date = DATA$date
BT_price = DATA$BT_price
T = length(BT_price)
ret = 100*(BT_price[2:T]-BT_price[1:(T-1)])/BT_price[1:(T-1)]
```

Now, we look at the autocorrelation function to see if we could exploit some autocorrelations to predict future returns.

```
autocorr = acf(ret)
```




```
(autocorr)
```

```
##
## Autocorrelations of series 'ret', by lag
##
```

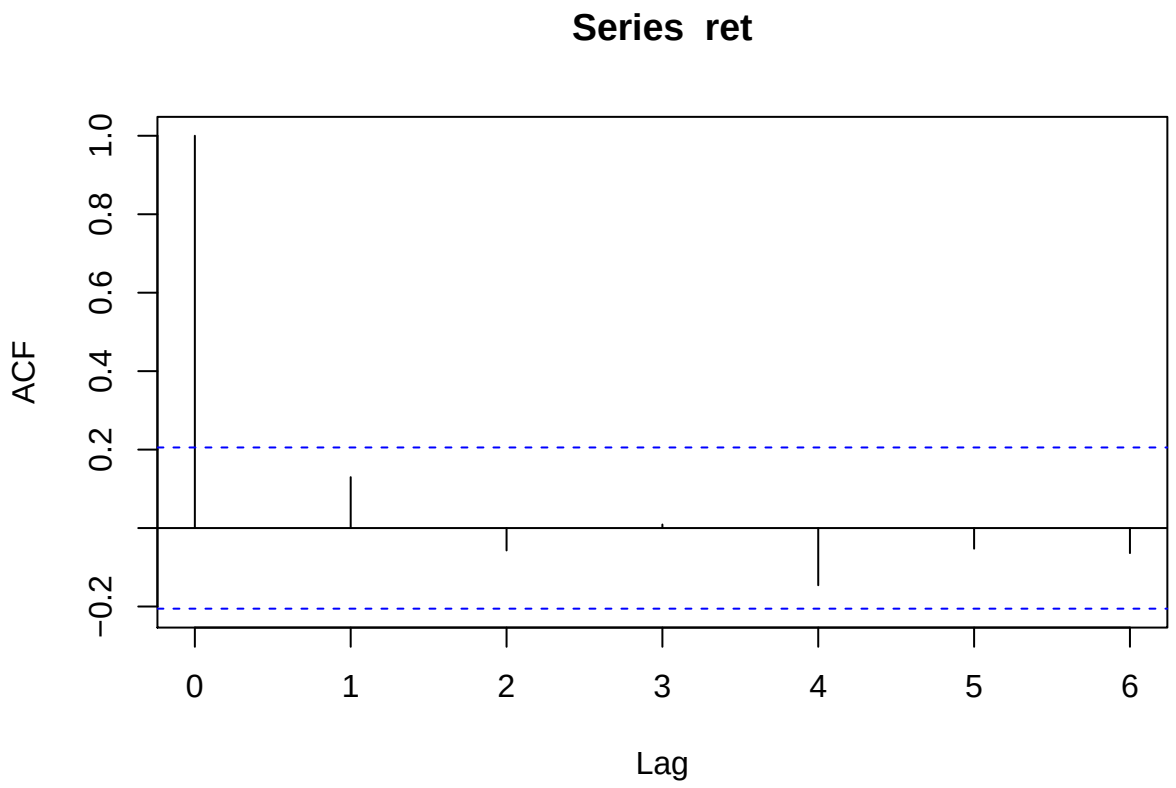
```
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.130 -0.057  0.009 -0.146 -0.053 -0.064  0.047 -0.002 -0.138 -0.019
##     11     12     13     14     15     16     17     18     19
## -0.036  0.132  0.043  0.068 -0.022 -0.057  0.035 -0.032  0.056
```

Unfortunately, it seems that no autocorrelation could be exploited (which is in line with the efficiency market hypothesis). We end by formally testing the presence of autocorrelation. To do so, we perform the Box-Pierce test.

```
Box.test(ret,lag = 5) ## R function
```

```
##
##  Box-Pierce test
##
## data:  ret
## X-squared = 4.0178, df = 5, p-value = 0.5469
```

```
## Performing manually the same test
max_lag = 6
acf_val = acf(ret,lag.max = max_lag)
```




```
val = acf_val$acf
T = length(ret)
T*sum(val[2:max_lag]^2)
```

```
## [1] 4.0178
```

```
qchisq(0.95,max_lag-1)
```

```
## [1] 11.0705
```

We do not reject the Null hypothesis of no-autocorrelation. In fact, the critical value amounts to 11.07 (at 5%) and the value of the test is equal to 4.01.