MA simulation

Simulation of a MA(1) process

In this short note, we show how to simulate from a MA(1) model. In particular, we shall simulate realizations from the model given by

$$y_t = \mu + \theta_1 \epsilon_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.t(v),$$

where $t(v)$ denotes the student distribution with v degree of freedom. Note that if $\epsilon_t \sim t(v)$, then $\sigma^2 = \text{Var}(\epsilon_t) = \frac{v}{v-2}$. The statistical properties of the model are as follows,

$$E(y_t) = \mu, \quad \text{Var}(y_t) = \sigma^2(1+\theta_1^2), \quad \text{Corr}(y_t, y_{t-1}) = \frac{\theta_1}{1+\theta_1^2}.$$

Let us check these results empirically.

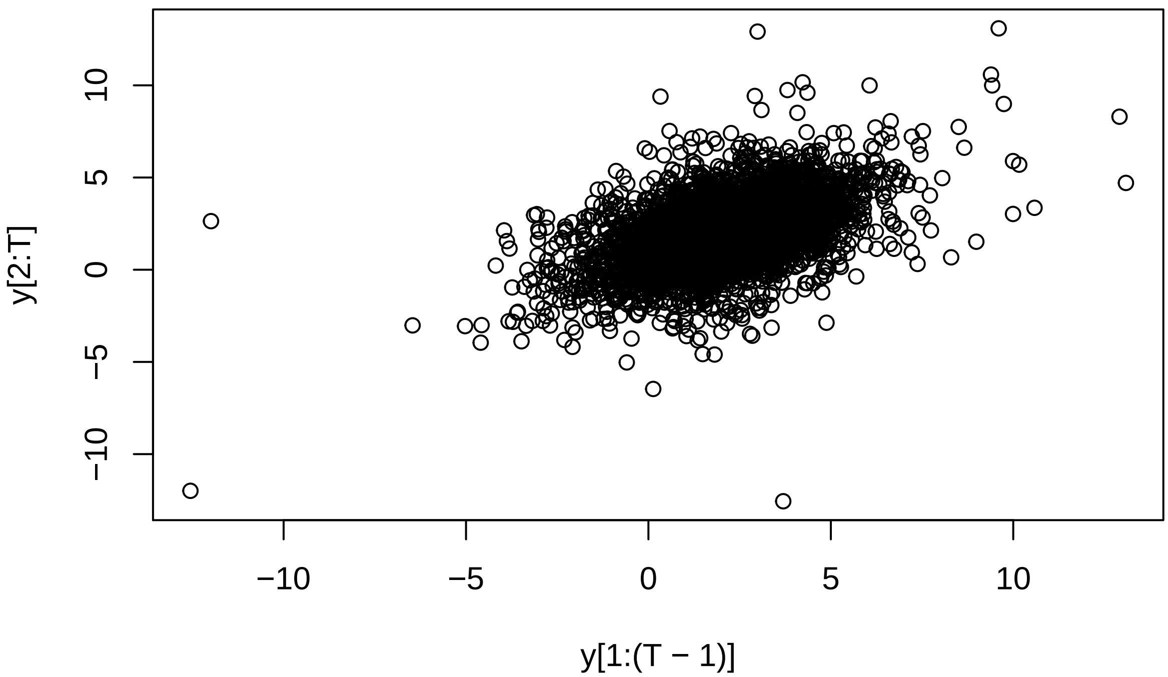
```
T = 5000 ## sample size
mu = 2
theta_1 = 0.9
df = 5
epsilon = rt(T,df)
y = numeric(T)
y[1] = mu
for(t in 2:T){
  y[t] = mu + theta_1*epsilon[t-1] + epsilon[t]
}
stat_th = c(mu,df*(1+theta_1^2)/(df-2),theta_1/(1+theta_1^2))

corr_emp = mean((y[2:T]-mean(y))*(y[1:(T-1)]-mean(y)))/var(y)
stat_emp = c(mean(y),var(y),corr_emp)

df_stat = data.frame(cbind(stat_th,stat_emp))
rownames(df_stat) = c("Expectation","Variance","Correlation")
colnames(df_stat) = c("Theoretical","Empirical")
(df_stat)
```

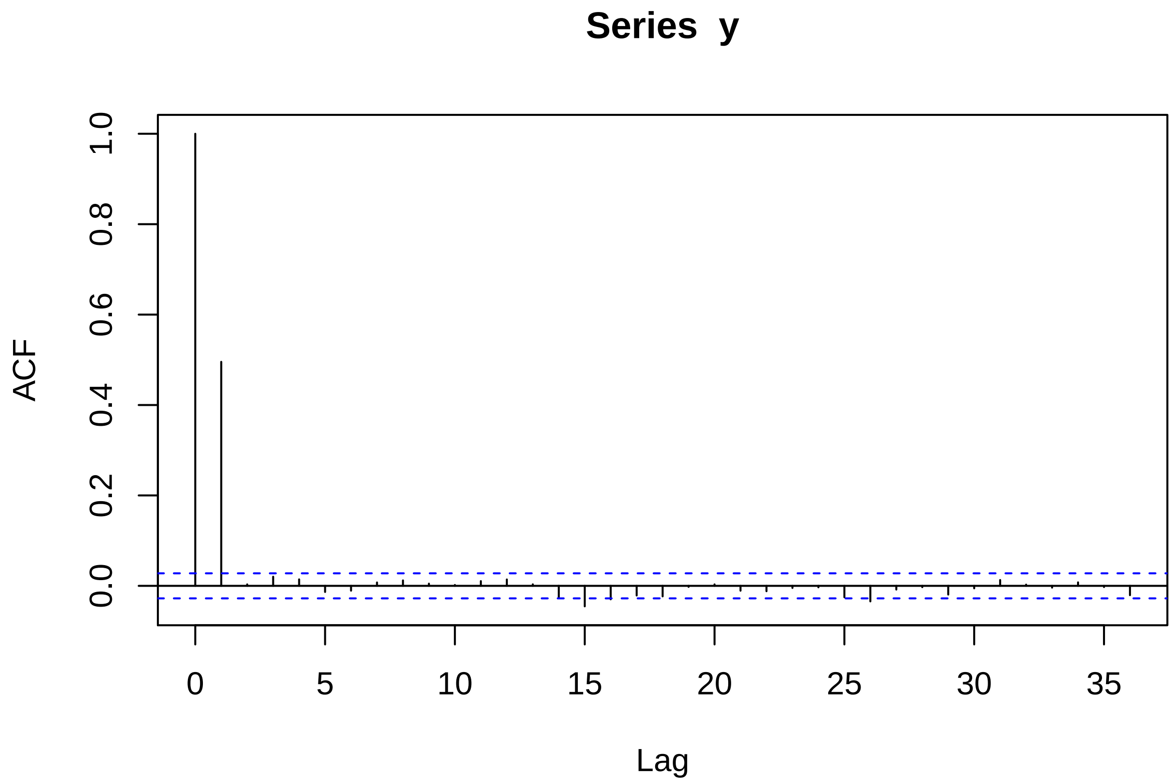
```
##             Theoretical Empirical
## Expectation   2.0000000 1.9820846
## Variance      3.0166667 3.0883824
## Correlation   0.4972376 0.4955353
```

```
plot(y[1:(T-1)],y[2:T])
```



We observe that the empirical statistics are close to the theoretical one. We conclude that we have correctly simulated the MA(1) process. We end this exercise by computing the autocorrelation function (ACF). Since the autocorrelation is equal to zero after the first lag, we should observe not significant empirical correlation after the first lag.

```
acf(y)
```


Series y
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
Lag